BOUNDARY-LAYER DEVELOPMENT IN THE INITIAL SECTION OF A TUBE WITH INJECTION

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Results are presented of a theoretical and experimental investigation of turbulent boundary-layer development in the initial section of a tube in the presence of injection. It is hence considered that there is no main flow. Formulas are derived to compute the friction coefficient and the dynamic characteristics of the flow in the hydrodynamic stabilization section for subsonic gas-motion velocities. The proposed method of computation is compared with the results of an experimental investigation.

The influence of injection on the characteristics of the main flow is studied in [1]. However, cases are encountered in practical applications when there is no main flow and the flow is limited either because of entrance of gas into the channel through its penetrable walls or because of evaporation or pitting of the walls themselves.

NOTATION

B – constant
b – permeability parameter
c_f – local friction coefficient
D – diameter
H – shape parameter
$(\rho w)_w$ – stream density
R – Reynolds number
R_w – penetrability factor
w – velocity
δ^* – displacement thickness
δ^{**} – momentum-loss thickness
M – dynamic-viscosity coefficient
t – time
ρ – density
γ – density of the wall material
x, l – length
τ – tangential stress
Ψ – relative friction coefficient
ω – relative velocity
ξ – relative coordinate
m, n – exponents
U – rate of change of the aggregate state

SUBSCRIPTS

0 – parameters on the outer boundary of the boundary layer
w – parameters at the wall

In the case under consideration it is convenient to write the system of initial equations in the form [2]:

momentum equation

$$\frac{dR^{**}}{dX} + (1 + H)\frac{R^{**}}{R_0}\frac{dR_0}{dX} = R_0\frac{c_{f_0}}{3}(\Psi + b) \tag{1}$$

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continuity equation

$$4HR^{**} = R_0 - 4\int_0^X R_w dX \tag{2}$$

relative friction coefficient

$$\Psi = \left(1 - \frac{b}{b_*}\right)^2, \; R^{**} = \frac{\rho_0 w_0 \delta^{**}}{\mu}, \; H = \frac{\delta^*}{\delta^{**}}, \; \frac{c_{f_0}}{2} = \frac{\tau_{w_0}}{\rho_0 w_0^2} \tag{3}$$

$$X = \frac{x}{D}, \; b = \frac{(\rho w)_w}{\rho_0 w_0}\frac{2}{c_{f_0}}, \; \Psi = \left(\frac{c_f}{c_{f_0}}\right)_{R^{**}}, \; R_w = \frac{(\rho w)_w D}{\mu}, \; R_0 = \frac{\rho_0 w_0 D}{\mu} \tag{4}$$

The system (1)-(4) can be solved for a known dependence of the shape factor H on the penetrability parameter b.

This dependence can be determined if relations obtained earlier in [2] are utilized:

$$\int_w^1 \frac{d\omega}{\sqrt{\Psi + b\omega}} = z, \quad z = 1 - \Omega, \quad \omega = \frac{w^*}{w_0} \tag{5}$$

Here $\Omega = \xi^n$ is the velocity profile in the turbulent nucleus of the boundary layer for isothermal gradient gas flow. Solving the results of integrating (5) with respect to ω, we obtain

$$\omega = 1 - (1 - \xi^n)\sqrt{\Psi + b} + (1 - \xi^x)^2 b/4 \tag{6}$$

The integral turbulent boundary-layer characteristics δ^*, δ^{**}, H are computed by means of the known velocity profile:

$$\delta^* = \int_0^\delta (1 - \omega)\left(1 - \frac{y}{r_0}\right) dy, \; \delta^{**} = \int_0^\delta \omega(1 - \omega)\left(1 - \frac{y}{r_0}\right) dy \tag{7}$$

As has been shown in [1], the results of a computation performed by means of (6) and (7) are satisfactorily approximated by the dependence

$$H = H_0(1 + kb) \tag{8}$$

Here H_0 is the value of the shape parameter under standard conditions.

It is convenient to write the friction law in the domain of subsonic gas-flow velocities as

$$c_{f_0} = BR^{**-m} \tag{9}$$

From (1)-(9) we then obtain a system of dynamic boundary-layer equations in the initial section of a tube in the presence of just a transverse flow of substance:

$$\frac{dR^{**}}{dX} + (1 + H)\frac{R^{**}}{R_0}\frac{dR_0}{dX} = R_0 \frac{B}{2}\frac{\Psi + b}{R^{**m}} \tag{10}$$

$$4HR^{**} = R_0 - 4\int_0^X R_w dX, \; b = \frac{2}{B}\frac{R_w}{R_0}R^{**m}, \; H = H_0(1 + kb) \tag{11}$$

Let us examine some particular cases of solving the system (10), (11). Let us put b = const. In this case the system (10)-(11) reduces to a linear first-order differential equation:

$$\left\{1 + H\frac{\Psi + b}{b}\right\}\frac{dR^{**}}{dR_0} + (1 + H)\frac{R^{**}}{R_0} - \frac{\Psi + b}{4b} = 0 \tag{12}$$

whose solution it is convenient to write as

$$R^{**} = cR_0, \qquad c = \frac{\Psi + b}{4\{b(2 + H) + H(\Psi + b)\}} \tag{13}$$

Substituting (13) into (10) and (11), we obtain the Reynolds number dependence:

$$R^{**} = dX, \; d = \frac{B}{2}\frac{m}{2 + H}\frac{\{4[b(2 + H) + H(\Psi + b)]\}^{m+1}}{(\Psi + b)^m} \tag{14}$$

and the penetrability factor as a function of the longitudinal coordinate:

$$R_w = \frac{B}{2}b\frac{R_0}{R^{**m}} \tag{15}$$

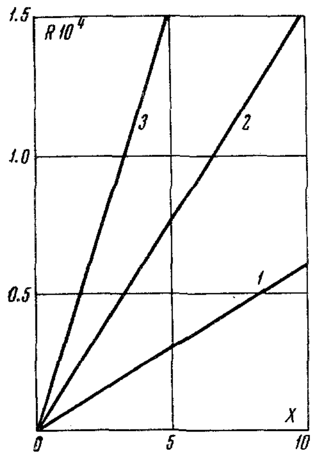

Fig. 1

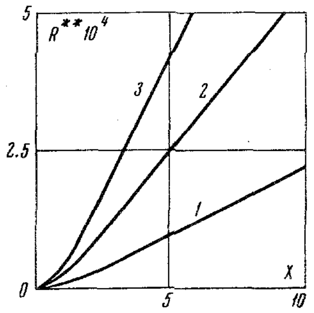

Fig. 2

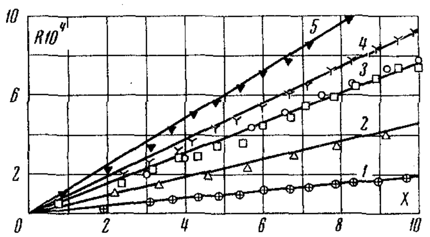

Fig. 3

Let us put $(\rho w)_W = \text{const}$. In this case the system reduces to a nonlinear differential equation of the form

$$1 + (1 + H)\frac{R^{**}}{R_0}\frac{dR_0}{dR^{**}} - \frac{\Psi + b}{4b}\left\{\frac{dR_0}{dR^{**}} - \frac{d\,(4HR^{**})}{dR^{**}}\right\} = 0 \tag{16}$$

which solved with respect to the derivative yields an equation convenient for integration by a numerical method:

$$\frac{dR_0}{dR^{**}} = \frac{4\,(H + H_0kmb + b/(\Psi + b)}{1 + 4H_0kb\,(R^{**}/R_0) - (1 + H)\,4bR^{**}\,/\,(\Psi + b)R_0} \tag{17}$$

$$X = \frac{R_0 - 4HR^{**}}{4R_w} \tag{18}$$

Figures 1 and 2 present the results of computing the Reynolds number constructed along the diameter and the thickness of the momentum loss as a function of the longitudinal coordinate for different values of the penetrability factor: $R_W = 1\text{-}10^4$, $2\text{-}2.5 \cdot 10^4$, $3\text{-}5 \cdot 10^4$. In this case (17) was solved numerically on the M-20 computer by the Runge-Kutta method for $B = 0.0256$, $k = 0.05$, $m = 0.25$. The computation was checked by dividing the integration spacing in half.

In practical applications it is often necessary to deal with the problems elucidated above, but the outer boundary of the channel hence changes in time by the mass of the channel wall going over into the gaseous state. In this case, we can write on the basis of the law of mass conservation

$$2\pi lr\,(\rho w)_w dt = \gamma_w 2\pi lr dr \tag{19}$$

From (19) follows

$$(\rho w)_w = \gamma_w \frac{dr}{dt} \tag{20}$$

Remarking that $dr/dt = U$ is the rate of change of the aggregate state, we have

$$(\rho \omega)_w = \gamma_w U \tag{21}$$

Integrating (20) with (21) taken into account, we obtain the law of the time change in the radius:

$$r = r_1 + Ut \tag{22}$$

and the connection between the flux density and the geometric channel characteristics:

$$(\rho w)_w = \frac{\gamma_w\,(r_2 - r_1)}{t_+} \tag{23}$$

Here r_2 and r_1 are the final and initial radii of the channel, respectively; t_+ is the total time of the process. Solving (22), (23) jointly, we obtain the connection between the penetrability factor and the external parameters of the process:

$$R_w = \frac{2\Upsilon_w U}{\mu}\{r_1 + U_t\} \tag{24}$$

Therefore, the system (9)-(11), (24) permits computation of all the necessary dynamical time characteristics of the flow.

An experimental investigation of the formulated problem was carried out on an apparatus whose diagram and description are presented in [1]. The forward endface of the porous tube was hence plugged up.

The total and static pressures along the tube axis as well as the flow temperature were measured during the test. The velocity and Reynolds number were computed by means of the measured parameters.

The range of variation of the flux density $(\rho w)_W$ in the experiments was 554-3600 $kg/m^2 \cdot h$. The results of tests 1-5, presented in Fig. 3 for values of R_W = 300, 788, 1290, 1533, 1945, are compared with the data of a numerical computation by means of (17) and (18). As is seen, satisfactory agreement between the method proposed and the experiment holds.

Therefore, the following sequence of a dynamic boundary-layer computation in the initial portion of the tube in the presence of injection can be proposed.

In the case $b = \text{const}$,

1) the parameters Ψ and H are determined for a given penetrability parameter from (3) and (11);

2) the Reynolds-number distribution is determined by (13) and (14) when the longitudinal coordinate X is given;

3) the distribution of the penetrability factor is computed by means of (15), and the change in the friction coefficient along the tube length by means of (3) and (8).

In the case $(\rho w)_W = \text{const}$,

1) the value of the penetrability factor R_W is determined by means of (4) and known D and μ;

2) having been given the number R_0 or R^{**}, the R^{**} or R_0 for given R_W is then determined numerically by means of (17);

3) Longitudinal coordinate X is computed by means of (18), and the penetrability parameter b by means of (11);

4) the distribution of the friction coefficient is found from (3) and (8). In the case of computing a variable-radius channel, the penetrability factor R_W is determined from (24) for a specific time, and the computation at $(\rho w)_W = \text{const}$ is performed for this value of R_W.

LITERATURE CITED

1. A. I. Leont'ev, A. V. Fafurin, and P. V. Nikitin, "Turbulent boundary layer in the initial portion of a tube under nonisothermal and injection conditions," Teplofiz. Vysok. Temp., 7, No. 2 (1969).
2. Heat and Mass Exchange and Friction in a Turbulent Boundary Layer [in Russian], Izd-vo SO AN SSSR, Novosibirsk (1962).